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RANDOM RECORD MODELS

by

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We study record times, mainly, and sizes in the following context. Let X_n denote the size of the n^{th} event occurring in a point stochastic pacing process, P the X_n is i.i.d., and P is, variously, Poisson, negative binomial, renewal, and Furry. Explicit distributions of first record times are (cont.

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found, domains of abstraction studied, and the asymptotic lognormality of the $n^{\underline{th}}$ record time is shown for Poisson P.

RANDOM RECORD MODELS

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1. Introduction.

Many situations suggest the study of the following class of models: a point stochastic pacing process, P, with counting process $\{N(t), t \geq 0\}$ develops over time, and associated with each event or "point" is a real-valued random variable, X; let X_n denote the n^{th} such random variable, realized at the moment $N(t)$ first reaches n. Here P may be interpreted as specifying the occurrence of demands--or supplies--of goods, water, payments or reimbursements, opportunities, etc., while X represents a generic demand or reward size, X_n being the magnitude of the n^{th} event. Alternatively P governs the occurrence of system shocks, possibly caused by floods or earthquakes, and X is a generic shock magnitude.

The objective of this paper is to introduce various simple models for P, and for $\{X_n\}$ and to study the occurrence of record events ("new highs") in the X-sequence as they appear under the stimulus of P. Related investigations are those into the classical record problem by Barton and Mallows [1], Chandler [2], Foster and Stuart [7], and Shorrock, e.g. [13], [14]. The studies of extremal processes by Dwass [4], [5], see also Resnick [12] and Resnick and Rubinovitch [11] also are pertinent. In particular

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Pickands, in [10], allows record events to be Poisson paced. The results reported in this paper are mainly explicit forms, and domains of attraction, for first record-time distributions when P is one of several familiar point processes (Poisson, renewal, pure birth). In addition we point out that our distributions may be interpreted as equipment lifetime distributions in the reliability sense, and that the classical "marriage problem" of sequential decision theory, cf. De Groot [3], may be viewed as a record problem.

2. Process Specification.

A point stochastic <u>pacing process</u>, P, with counting process $\{N(t), t \geq 0\}$ governs the occurrence of <u>values</u>, denoted by $\{X_n, n=0,1,2,\ldots\}$. Assume P and $\{X_n\}$ to be mutually independent, and the $\{X_n\}$ to be an i.i.d. sequence with absolutely continuous d.f. $F(x)$. Mainly we shall assume that P is an orderly process, so we can speak of the times between successive events in P; let the n^{th} of these be $\{T_n, n=1,2,\ldots\}$ so that the time of the n^{th} process event is $t_n = \sum_{i=1}^{n} T_i$, $n = 1,2,\ldots$ Suppose that at $t = 0$ a reference value, X_0, is available; X_0 is a random variable with d.f. $F_0(x)$.

<u>Definition</u>. A first record with respect to X_0 occurs at $t = t_{n_1}$ if (a) $X_i < X_0$, all $i < n_1$ (b) $X_{n_1} > X_0$, in which case $\tau_1 = t_{n_1}$ = the first record time, and $X_{n_1} = R_1$, the first record value. Subsequently, a k^{th} record occurs at time $t_{n_k} = \sum_{i=1}^{k} \tau_i$ if (a) $X_i < R_{k-1}$, $n_{k-1} < i < n_k$, (b) $X_{n_k} > R_{k-1}$, and $\tau_k = t_{n_k} - t_{n_{k-1}}$, $R_k = X_{n_k}$. Alternatively

$$\tau_1 = \inf\{t: \max_{n \leq N(t)} X_n > X_0\} \tag{2.1}$$

$$R_1 = X_{N(\tau_1)}, \tag{2.2}$$

$$\tau_1 + \tau_2 = \inf\{t: \max_{n \leq N(t)} X_n > R_1\}$$

$$R_2 = X_{N(\tau_1 + \tau_2)}, \ldots .$$

3. Poisson Paced First Records, with Variations.

Suppose P is Poisson with time-dependent rate $\lambda(t)$, and hazard $\Lambda(t) = \int_0^t \lambda(t')dt'$. If $F_0(\cdot)$ denotes the d.f. of X_0, and $F(\cdot)$ the d.f. of $\{X_n, n=1,2,\ldots\}$, then

$$P\{\tau_{(1)} \varepsilon (dt), R_1 \varepsilon (dz)\} = \int_{\infty}^{z} e^{-\Lambda(t)[1-F(x)]} dF_0(x) d\Lambda(t) dF(z) \qquad (3.1)$$

from which various special cases of interest immediately appear.

3.1 $F_0(\cdot) = F(\cdot)$.

In this case the integration is immediate, and

$$P\{\tau_1 \varepsilon (dt), R_1 \varepsilon (dz)\} = \frac{e^{-\Lambda(t)[1-F(z)]} - e^{-\Lambda(t)}}{\Lambda(t)} d\Lambda(t) dF(z). \qquad (3.2)$$

The marginal d.f. of record size is obtained by integrating out t in (3.1); the density is

$$P\{R_1 \varepsilon (dz)\} = -\log[1-F(z)]dF(z) \qquad (3.3)$$

just as in the classical setup. The marginal d.f. of τ_1 is given by integrating first on z. Most simply, however,

$$P\{\tau_1 > t \mid X_0 = x\} = e^{-\Lambda(t)[1-F(x)]}; \qquad (3.4)$$

Removal of the condition using $F_0 = F$ yields

$$P\{\tau_1 > t\} = \frac{1 - e^{-\Lambda(t)}}{\Lambda(t)} . \qquad (3.5)$$

Note that in this, and in other, cases $P\{\tau_1 > t\}$ is given by integration of the generating function of the counting process of P:

$$P\{\tau_1 > t\} = \int_{-\infty}^{\infty} \sum_{n=0}^{\infty} e^{-\Lambda(t)} \frac{[\Lambda(t)]^n}{n!} F^n(x) dF(x). \qquad (3.6)$$

Further note that since $\Lambda(t)$ is non-decreasing, then if $h > 0$

$$E[\tau_1] = \int_0^\infty P\{\tau_1 > t\}dt = \int_0^h \frac{1-e^{-\Lambda(t)}}{\Lambda(t)}\,dt + \int_h^\infty \frac{1-e^{-\Lambda(t)}}{\Lambda(t)}\,dt,$$

$$(1-e^{-\Lambda(h)})\int_h^\infty \frac{dt}{\Lambda(t)} \le E[\tau_1] \le h + \int_h^\infty \frac{dt}{\Lambda(t)}$$

so

$$E[\tau_1] < \infty \quad \text{iff} \int_h^\infty \frac{dt}{\Lambda(t)} < \infty \quad \forall\ h > 0. \tag{3.7}$$

For time homogeneous Poisson P it is clear that no finite moments exist.

3.2 Random (Gamma) Hazard, $F_0(\cdot) = F(\cdot)$.

If $\{\Lambda(t)\}$ is a gamma process, then $E[e^{-s\Lambda(t)}] = (\frac{\mu}{\mu+s})^{kt}$, and $\{N(t)\}$ is a negative binomial process. Therefore

$$P\{\tau_1 > t\} = \int_{-\infty}^\infty \left[\frac{\mu}{\mu+1-F(x)}\right]^{kt} dF(x) = \frac{\mu}{1-kt}\left[\left(1+\frac{1}{\mu}\right)^{1-kt} - 1\right] \tag{3.8}$$

and

$$P\{\tau_1 > t\} \sim \frac{\mu}{kt} = \frac{1}{E[\Lambda(t)]} \quad \text{as} \quad t \to \infty. \tag{3.9}$$

Letting k decrease but such that $\frac{k}{\mu} = 1$ induces an increasingly long Pareto tail, and a buildup of mass near zero. A similar effect is achieved by mixing Poisson P's rate parameter by the gamma (applying a gamma prior to λ):

$$P\{\tau_1 > t\} = \int_0^\infty \frac{1-e^{-\lambda t}}{\lambda t}\, e^{-\mu\lambda}\, \frac{(\mu\lambda)^{k-1}}{\Gamma(k)}\,\mu d\lambda = \frac{\mu}{t}\,\frac{\left(1+\frac{t}{\mu}\right)^{-k+1} - 1}{-k+1} \tag{3.10}$$

If P has been observed throughout the time interval $(-T,0)$, and j events occurred, then the posterior is gamma with $\mu = T+a$, $k = j+b$, where the prior density is $p(\lambda) = e^{-\lambda a}\frac{(\lambda a)^{b-1}}{\Gamma(b)}\,a$ and (3.6) provides a Bayes-type predictor of the distribution of time until the first record, the latter being referred to an initial X_0 with d.f. F.

3.3 Process Observed for Initial Time T.

If the P-paced process has been observed for $(-T,0)$ and further records are referred to the largest X occurring therein, then

$$F_0(x) = e^{-\Lambda(T)[1-F(x)]}, \tag{3.11}$$

and application to (3.4) yields

$$P\{\tau_1 > t\} = \frac{\Lambda(T)}{\Lambda(T)+\Lambda(t)}\{1 - e^{-[\Lambda(T)+\Lambda(t)]}\}; \tag{3.12}$$

with <u>confidence</u> given by the r.h.s. of (3.12) there will be no new maximum in $(0,t)$ greater than that in $(-T,0)$.

3.4 The Distribution of τ_n.

Since τ_n, the time between $n-1^{\underline{st}}$ and $n^{\underline{th}}$ records does not depend upon the distribution of X, we may capitalize upon the Markovian property of the exponential, $F(x) = 1 - \exp(-x)$, to characterize the distribution of τ_n for any n. Now for the time-homogeneous Poisson and exponential X,

$$P\{\tau_n \varepsilon (dt) | R_{n-1}=x\} = e^{-\lambda t e^{-x}} \lambda e^{-x} dt. \tag{3.13}$$

We can write the characteristic function (ch.f.) of log τ_n as follows. First,

$$E[e^{iz\log\tau_n}|R_{n-1}=x] = \int_0^\infty t^{iz} e^{-\lambda t e^{-x}} \lambda e^{-x}\,dx$$
$$= \left(\frac{1}{\lambda}\right)^{iz} e^{izx} \int_0^\infty u^{iz} e^{-u}\,du. \qquad (3.14)$$

Next remove the condition using the fact that R_{n-1} is gamma:

$$E[e^{iz\log\tau_n}] = \left(\frac{1}{\lambda}\right)^{iz} \int_0^\infty e^{izx} e^{-x} \frac{x^{n-1}}{(n-1)!}\,dx \int_0^\infty u^{iz} e^{-u}\,du$$
$$= e^{iz\log\lambda}\left(\frac{1}{1-iz}\right)^n \int_0^\infty e^{iz\log u} e^{-u}\,du, \qquad (3.15)$$

which may, on the basis of ch.f. unicity, be interpreted as saying that the distribution of $\lambda\tau_n$ is that of the sum of two independent random variables, one being gamma with mean and variance equalling n, and the other being the logarithm of a unit exponential. Moreover, the ch.f. of $\frac{1}{n}\log\lambda\tau_n$ is

$$E[e^{iz\frac{\log\lambda\tau_n}{n}}] = \left(1-\frac{iz}{n}\right)^{-n} \int_0^\infty e^{\frac{iz}{n}\log u} e^{-u}\,du \qquad (3.16)$$

$$\to e^{iz} \qquad \text{as} \qquad n\to\infty$$

and invocation of the continuity theorem for the ch.f. shows that $\sqrt[n]{\lambda\tau_n} \to e$ in probability. Finally, consider the ch.f. of $\dfrac{\log\lambda\tau_n - n}{\sqrt{n}}$. As n becomes large the gamma component dominates and the continuity theorem shows that the latter sequence of r.v. tends to $N(0,1)$ in law. These results are analogous to those of Neuts [9] for the classical record problem.

4. Renewal-Paced Records.

Suppose the pacing process P is an arbitrary renewal process with i.i.d. interevent times $\{U_n\}$ having d.f. $H(\cdot)$, and $H(0+) = 0$. Then

$$P\{\tau_1 > t \mid X_0 = x\} = \sum_{n=0}^{\infty} [H^{n*}(t) - H^{(n+1)*}(t)]F^n(x) \tag{4.1}$$

and, Laplace transforming, the following explicit form appears:

$$\int_0^\infty e^{-st} P\{\tau_1 > t \mid X_0 = x\}dt = \frac{1-\hat{H}(s)}{s} \frac{1}{1-\hat{H}(s)F(x)} \tag{4.2}$$

where $\hat{H}$ is the Laplace-Stieltjes transform of H. If $F_0 = F$, we integrate to obtain

$$\int_0^\infty e^{-st} P\{\tau_1 > t\}dt = \frac{1-\hat{H}(s)}{-s\hat{H}(s)} \log[1-\hat{H}(s)], \tag{4.3}$$

so

$$E[e^{-s\tau_1}] = 1 + \frac{1-\hat{H}(s)}{\hat{H}(s)} \log[1-\hat{H}(s)]. \tag{4.4}$$

Example 1. If P is time homogeneous Poisson we have seen that $P\{\tau_1 > t\} \sim \frac{k}{\lambda t}$ as $t \to \infty$, just as is true when $H(t) = \begin{cases} 1 & \text{for } t \geq 1/\lambda \\ 0 & \text{for } t < 1/\lambda, \end{cases}$

i.e. when records are regularly paced.

Example 2. If H is a stable distribution, then $\hat{H}(s) = e^{-s^\alpha}$ $(0 < \alpha < 1)$ and this and (4.3) yield

$$\int_0^\infty e^{-st} P\{\tau_1 > t\} = \frac{e^{s^\alpha}-1}{s} \log[1-e^{-s^\alpha}] \sim -\alpha s^{\alpha-1} \log s. \tag{4.5}$$

so by a Tauberian theorem (Feller [6], p. 447),

$$P\{\tau_1 > t\} \sim \frac{\alpha}{\Gamma(1-\alpha)} t^{-\alpha}\log t \qquad (t\to\infty) \tag{4.6}$$

For instance if $\alpha = \frac{1}{2}$ then H tails off like $t^{-1/2}$ while the d.f. of τ_1 tails off like $t^{-1/2} \log t$.

4.1 Domains of Attraction.

Suppose that n independent copies of τ_1 accumulated and summed: $S_n = \sum_{i=1}^{n} \tau_1(i)$. We record several facts about the behavior of S_n for large n ascertainable from general theory; see Feller, II [6], and Gnedenko and Kolmogorov [8], p. 175, Theorem 2.

A. If $P\{\tau_a > t\} \sim \frac{K}{\lambda t}$, as is true when P is Poisson, regular-spaced, and in other situations as well, S_n is attracted to a stable law of order $\alpha = 1$. Direct expansion of the relevant Laplace transform around $s = 0$ shows that $S_n / n \log n \to 1$ in probability.

B. If P is renewal with stable law of order α $(0<\alpha<1)$ interevent times, then because of the asymptotic behavior of 4.6), the further results of Feller [6], pp. 448-449 immediately imply that

$$\frac{S_n}{a_n} \overset{(d)}{\to} Z_\alpha \tag{4.7}$$

Z_α being <u>once again stable of order α</u>. Here a_n satisfies

$$a_n^\alpha \sim \alpha n \log a_n \tag{4.8}$$

and $a_n > n^{1/\alpha}$ for large n: obviously a strong norming is required to bring the stable-paced record times back to stable. A suitable normalizing constant derived from (4.8) is seen to be

$$a_n \sim (n \log n)^{1/\alpha}; \tag{4.9}$$

This result also follows immediately from a transform continuity theorem. Simply scale by a_n, $(a_n \to \infty)$.

$$\left(E\left[e^{-\frac{s}{a_n}\tau_1}\right]\right)^n = \{1 + [\exp\left(\frac{s}{a_n}\right)^{\alpha} - 1]\log[1 - \exp(-\left(\frac{s}{a_n}\right)^{\alpha}]\}^n \tag{4.10}$$

and expand in Taylor's series to see that if $\frac{n \log a_n^{\alpha}}{a_n^{\alpha}} \to 1$ then the above (4.10) tends to $e^{-s^{\alpha}}$, the stable law (α) transform.

5. Furry Records.

For variety let P be the simple pure-birth or Furry-Yule process ([6], I, p. 450) generated by $\lambda_n = n\lambda$. Start with a single progenitor at $t = 0$ (of value X_0 drawn from $F(x)$). Now $N(t)$, the number of new births in $(0,t)$, is geometric, and each is endowed with a value, X, that is an independent copy of X_0. Hence

$$g(z,t) = E[z^{N(t)}] = \frac{e^{-\lambda t}}{1-z(1-e^{-\lambda t})} \tag{5.1}$$

and

$$P\{\tau_1 > t\} = \int_0^1 g(z,t)dz = \int_{-\infty}^{\infty} \frac{e^{-\lambda t} \, dF(x)}{1-F(x)[1-e^{-\lambda t}]}$$

$$= \frac{\lambda t e^{-\lambda t}}{1-e^{-\lambda t}} = \frac{\lambda t}{e^{\lambda t}-1}$$

$$= \sum_{k=0}^{\infty} \frac{B_i}{i!} (\lambda t)^i, \tag{5.2}$$

where the B_i are the Bernoulli numbers. Integration of (5.2) yields the mean of the first record time:

$$E[\tau_1] = \lambda^{-1}[1 + \frac{1}{2^2} + \frac{1}{3^2} + \ldots] = \frac{1}{\lambda} \frac{\pi^2}{6} \simeq \frac{1.65}{\lambda} . \tag{5.3}$$

Thus the Furry P process encourages a record to occur after a mean delay of very nearly half way between the mean second and third Furry jump times.

5.1 General Pure-Birth P; Laplace Transform.

If P is pure-birth with general intensity λ_n we find by following a recurrent events argument and an integration that

$$\int_0^\infty e^{-st} P\{\tau_1 > t\} dt = \sum_{n=1}^{\infty} \frac{1}{n\lambda_n} \prod_{j=1}^{n} \frac{\lambda_j}{\lambda_j + s} . \tag{5.4}$$

Now apply a Tauberian theorem allowing $s \to 0$ to obtain

$$E[\tau_1] = \sum_{n=1}^{\infty} \frac{1}{n\lambda_n} , \tag{5.5}$$

which shows that the first record time possesses a mean iff the series on the r.h.s. converges.

6. Models.

The P-paced record process suggests several specific models.

6.1 A Shock Model in Reliability.

Suppose X_n represents the magnitude of the n^{th} shock delivered by P. Let X_0 represent the "strength" of a component manufactured to inhabit the P environment; the component survives to time t if no shock in time t exceeds X_0. Hence (3.5), (3.8), (3.10), and (3.12) all may be interpreted as component life distributions. It may be reasonable to replace F_{X_0} by $[F(x)]^{k+1}$, $k > 0$, arguing that the manufacturer will attempt to build in a safety factor; safety increases with increasing k. In this case (3.2) becomes

$$P\{\tau_1>t\} = (k+1)\int_0^1 e^{-\Lambda(t)w}(1-w)^k dw \tag{6.1}$$

Example. If $F(x)$ is taken to be exponential with parameter θ, so $E[X] = \theta^{-1}$ then, having observed an X we can consider setting $X_0 = r^{-1}X$, $r < 1$ representing a safety factor. Then

$$P\{X_0\le x\} = 1 - e^{-\theta x} \tag{6.2}$$

and

$$P\{\tau_1>t\} = \int_0^\infty e^{-\lambda t[e^{-\theta x}]} r\theta e^{-r\theta x}dx$$

$$= \frac{\Gamma(r+1)}{(\lambda t)^r}\int_0^{\lambda t} \frac{e^{-z}z^{r-1}}{\Gamma(z)}\, dz \sim \frac{\Gamma(r+1)}{(\lambda t)^r} \tag{6.3}$$

as $t \to \infty$, and an exceedingly long tail results if $r < 1$, representing conservative design. The remarks of Section 4 indicate that sums of times to failure are attracted to the stable law of order r if $0 < r < 1$.

6.2 The Marriage Problem.

At the moments of event occurrence in a Poisson process a decision maker (art collector, suitor, employer, or what have you) is shown objects of value X. Allowed to inspect for only a finite time, and thwarted from any return to previous opportunities, he seeks to select that X of maximum value to occur in the interval.

Suppose the decision maker sets aside a time T to look over the field, leaving himself U for choice; and enjoyment or regret. $U + T$ is fixed. Let his decision rule be to examine all X-values to occur in T, and then to select the first X-record, if any, thereafter. He wishes to choose U in such a way as to maximize his probability of ending up with the greatest X-value to occur during $U + T$. Following this rule enables him to select the best X to occur throughout $(0,U+T)$ with probability nearly $e^{-1} = 0.368...$, exactly as in the classical formulation, cf. De Groot [3], in the event that opportunities come thick and fast, i.e. as $\lambda \to \infty$. For suppose he confronts the time period $(T,T+U)$, committed to picking the first record therein, if any occurs. Given x was the record during $(0,T)$, the probability that the first record to occur subsequently during $(T,U+T)$ is also the last is

$$\varphi(U,x) = \int_0^U e^{-\lambda t[1-F(x)]}\lambda dt \int_x^\infty dF(y) e^{-(U-t)[1-F(y)]}$$

$$= \int_0^U \frac{e^{-\lambda t[1-F(x)]} - e^{-\lambda U[1-F(x)]}}{U - t}\, dt. \tag{6.4}$$

The best opportunity during $(0,T)$ has distribution

$$F_0(x,T) = e^{-\lambda T[1-F(x)]} \tag{6.5}$$

and so the probability that the best overall is picked is

$$\varphi_\lambda(T) = \int_{-\infty}^{\infty} \varphi(U,x)\, dF_0(x,T) \tag{6.6}$$

Now interchange integration order and invoke (3.12) to obtain

$$\varphi_\lambda(T) = \int_0^U \frac{dt}{U-t} \left\{ \frac{T}{T+t} (1-e^{-\lambda(T+t)}) - \frac{T}{T+U} (1-e^{-\lambda(T+U)}) \right\} \tag{6.7}$$

If $\lambda \to \infty$ the integral tends (bounded convergence) to

$$\varphi(T) = \frac{T}{T+U} \int_0^U \frac{dt}{T+t} = \frac{T}{T+U} \log \frac{T+U}{T}$$

$$= -T \log T \qquad \text{if} \quad T + U = 1, \tag{6.8}$$

and differentiation shows that φ is maximized for $T = e^{-1}$. Finally, $\varphi(e^{-1}) = e^{-1}$, agreeing with the classical solution, see De Groot [3], p. 331. Now if the decision maker values the time to enjoy his prize as well as its attainment then he may be tempted to optimize $\max(U-\tau_1, 0)$, τ_1 being the time of the first and only record to occur after T, measured from T as origin.

Then his expected enjoyment is

$$\psi(T) = \frac{T}{T+U}\int_0^U \frac{(U-t)}{T+t}\,dt = T\log\left(\frac{T+U}{T}\right) - \frac{UT}{T+U}$$

$$= T\log\left(\frac{T+U}{T}\right) - \frac{UT}{T+U}$$

$$= -T\log T - T(1-T) \qquad (6.9)$$

with maximizing value close to $T = 0.2$, and $\psi(0.2) = 0.16$. If our decision maker chooses the latter strategy he wins with approximate probability 0.32 rather than the 0.37 probability of success achieved under the classical decision rule. On the other hand, if he chooses to observe for $T = e^{-1}$ his payoff under the time-weighted utility is 0.14. Perhaps $T = 0.25$ will appeal as an easily remembered compromise that nearly maximizes both goals mentioned. Finally, if λ becomes small it may be seen that optimum $T \to 0.5$ if the above rule is followed. However, this rule cannot be optimum under such light traffic situations: for very small λ one does best by choosing the first opportunity to appear.

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